

UNCERTAINTY BUDGET TABLES

AIR DENSITY

SOP 2 BUOYANCY CORRECTIONS (NISTIR 6969)

$$MABC = \Delta\rho\Delta V$$

$$u_{bc}^2 = \left(m_s \times \frac{\rho_s - \rho_x}{\rho_s \rho_x} \times u_{\rho_a} \right)^2 + [m_s \times (\rho_a - \rho_n)]^2 \times \left(\frac{u_{\rho_s}^2}{\rho_s^4} - \frac{u_{\rho_x}^2}{\rho_x^4} \right)$$

Component	Description	Reference
u_{ρ_a}	Standard uncertainty for air density	
u_e	Standard uncertainty for air density equation	CIPM Air density paper, 0.0012 mg/cm ³
u_t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.5 °C; SOP 2 (NISTIR 6969)
u_{rh}	Standard uncertainty for relative humidity	HB 143 accuracy guideline, 5 %; SOP 2 (NISTIR 6969)
u_p	Standard uncertainty for pressure	HB 143 accuracy guideline, 65 Pa; SOP 2 (NISTIR 6969)
u_{ρ_s}	Standard uncertainty for density of standard (ρ_s)	Calibration report, e.g., 0.008 g/cm ³
u_{ρ_x}	Standard uncertainty for density of unknown (ρ_x)	Mfg. Estimate, 0.05 g/cm ³

MASS

SOP 3 (DOUBLE SUBSTITUTION, EQUAL ARM) (NISTIR 6969)

SOP 4 (DOUBLE SUBSTITUTION) (NISTIR 6969)

SOP 5 (THREE IN ONE) (NISTIR 6969)

SOP 6 (TRANSPOSITION) (NISTIR 6969)

SOP 7 (SINGLE SUBSTITUTION) (NISTIR 6969)

$$C_x = C_s + d + \text{disparity (bias/drift) + unrelated corrections}$$

$$M_x = M_s + d + \rho_a (V_x - V_s) + \text{disparity (bias/drift) + unrelated corrections}$$

Component	Description	Reference
u_s	Standard uncertainty for standards	Calibration report, divide by k
s_p	Standard uncertainty for the process	Measurement assurance process; control charts
u_{bc}	Standard uncertainty for buoyancy correction	
u_{ρ_a}	Standard uncertainty for air density	
u_e	Standard uncertainty for air density equation	CIPM Air density paper (Davis/Giacomo, 1992), 0.0012 mg/cm ³
u_t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.5 °C; SOP 2 (NISTIR 6969)
u_{rh}	Standard uncertainty for relative humidity	HB 143 accuracy guideline, 5 %; SOP 2 (NISTIR 6969)
u_p	Standard uncertainty for pressure	HB 143 accuracy guideline, 65 Pa; SOP 2 (NISTIR 6969)
u_{ρ_s}	Standard uncertainty for density of standard (ρ_s)	Calibration report, e.g., 0.008 g/cm ³
u_{ρ_x}	Standard uncertainty for density of unknown (ρ_x)	Mfg. Estimate, 0.05 g/cm ³
u_d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
u_{ba}	Standard uncertainty of the balance	Typically not used if s_p is used.
u_{sens}	Standard uncertainty due to sensitivity of balance	OIML R111-1
u_{resol}	Standard uncertainty due to digital resolution of balance	OIML R111-1
$u_{ecc \text{ load}}$	Standard uncertainty due to eccentric loading	OIML R111-1
u_{mag}	Standard uncertainty due to magnetism	OIML R111-1

u_o	Standard uncertainty for other factors	
-------	--	--

SOP 8 (MODIFIED SUBSTITUTION) (NISTIR 6969)

$$C_x = C_s + d + \text{disparity (bias/drift)} + \text{unrelated corrections}$$

Component	Description	Reference
u_s	Standard uncertainty for standards	Calibration report, divide by k
s_p	Standard uncertainty for the process	Measurement assurance process
u_{bc}	Standard uncertainty for buoyancy correction	Possibly: uncorrected systematic error
u_{ρ_a}	Standard uncertainty for air density	
u_e	Standard uncertainty for air density equation	CIPM Air density paper (Davis/Giacomo, 1992), 0.0012 mg/cm ³
u_t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.5 °C; SOP 2 (NISTIR 6969)
u_{rh}	Standard uncertainty for relative humidity	HB 143 accuracy guideline, 5 %; SOP 2 (NISTIR 6969)
u_p	Standard uncertainty for pressure	HB 143 accuracy guideline, 65 Pa; SOP 2 (NISTIR 6969)
u_{ρ_s}	Standard uncertainty for density of standard (ρ_s)	Calibration report, e.g., 0.008 g/cm ³
u_{ρ_x}	Standard uncertainty for density of unknown (ρ_x)	Mfg. Estimate, 0.05 g/cm ³
u_d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
u_{ba}	Standard uncertainty of the balance	Typically not used if s_p is used.
u_{sens}	Standard uncertainty due to sensitivity of balance	OIML R111
u_{resol}	Standard uncertainty due to digital resolution of balance	OIML R111
$u_{ecc\ load}$	Standard uncertainty due to eccentric loading	OIML R111
u_{mag}	Standard uncertainty due to magnetism	OIML R111
u_o	Standard uncertainty for other factors	

**QUESTION: CAN u_{bc} BE ASSUMED TO BE NEGLIGIBLE, AT THIS LEVEL?
OR IS IT SAFER TO EVALUATE IT AND THEN DECIDE THE
NEGLIGIBILITY OF IT AND HAVE IT DOCUMENTED?**

SOP 28 (ADVANCED WEIGHING DESIGNS) (NISTIR 5672)

$$M_x = M_s + d + \rho_a (V_x - V_s) + \text{disparity (bias/drift)} + \text{unrelated corrections}$$

Component	Description	Reference
u_s	Standard uncertainty for standards	Calibration report, divide by k
s_p	Standard uncertainty for the process	Measurement assurance process; control charts
u_{bc}	Standard uncertainty for buoyancy correction	
u_{ρ_a}	Standard uncertainty for air density	
u_e	Standard uncertainty for air density equation	CIPM Air density paper (Davis/Giacomo, 1992), 0.0012 mg/cm ³
u_t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.5 °C; SOP 2 (NISTIR 6969)
u_{rh}	Standard uncertainty for relative humidity	HB 143 accuracy guideline, 5 %; SOP 2 (NISTIR 6969)
u_p	Standard uncertainty for pressure	HB 143 accuracy guideline, 65 Pa; SOP 2 (NISTIR 6969)
u_{CO_2}	Standard uncertainty for carbon dioxide determination	
u_{cee}	Standard uncertainty for cubical coefficient of expansion	
u_{ρ_s}	Standard uncertainty for density of standard (ρ_s)	Calibration report, e.g., 0.008 g/cm ³
u_{ρ_x}	Standard uncertainty for density of unknown (ρ_x)	Mfg. Estimate, 0.05 g/cm ³
u_d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
u_{ba}	Standard uncertainty of the balance	Typically not used if s_p is used.
u_{sens}	Standard uncertainty due to sensitivity of balance	OIML R111
u_{resol}	Standard uncertainty due to digital resolution of balance	OIML R111
$u_{ecc\ load}$	Standard uncertainty due to eccentric loading	OIML R111
u_{mag}	Standard uncertainty due to magnetism	OIML R111
u_o	Standard uncertainty for other factors	

LENGTH

SOP 10 (RIGID RULE) (NBS Handbook 145)

$$L_x = L_s + d + \text{disparity (bias/drift) + unrelated corrections}$$

Component	Description	Reference
u_s	Standard uncertainty for standards	Calibration report, divide by k
s_p	Standard uncertainty for the process	Measurement assurance process; range charts
u_{gr}	Standard uncertainty for graduated reticle	
u_d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
u_{res}	Standard uncertainty due to resetting of the rules	
u_o	Standard uncertainty for other factors	

SOP 11 (BENCH METHOD) (NBS Handbook 145)

SOP 23 (PI TAPES) (NBS Handbook 145)

$$L_x = L_s + d + L_n [(t - 20) (\alpha - \beta)] + \text{disparity (bias/drift) + unrelated corrections}$$

Component	Description	Reference
u_s	Standard uncertainty for standards	Calibration report, divide by k
s_p	Standard uncertainty for the process	Measurement assurance process; range charts
u_{gr}	Standard uncertainty for graduated reticle	
u_{tc}	Standard uncertainty for temperature correction	
u_t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.05 °C
$u_{lce (\alpha, \beta)}$	Standard uncertainty for linear coefficient of expansion	
u_{tw}	Standard uncertainty for tension weights	
u_d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
u_{res}	Standard uncertainty due to resetting of the tape	
u_o	Standard uncertainty for other factors	

SOP 12 (TAPE TO TAPE)

$$L_x = L_s + d + \text{disparity (bias/drift)} + \text{unrelated corrections}$$

Component	Description	Reference
u_s	Standard uncertainty for standards	Calibration report, divide by k
s_p	Standard uncertainty for the process	Measurement assurance process
u_{gp}	Standard uncertainty for graph paper	
u_{ss}	Standard uncertainty for spring scales	
u_d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
u_{res}	Standard uncertainty due to resetting of the tapes	
u_o	Standard uncertainty for other factors	

VOLUME - GRAVIMETRIC

SOP 13 (MECHANICAL BALANCE)

SOP 14 (ELECTRONIC BALANCE)

SOP 15 (EQUAL ARM BALANCE)

$$V_{t_{ref}} = (M_F - M_E)(M_S / M_R) \left(1 - \frac{\rho_a}{\rho_s} \right) \left(\frac{1}{\rho_w - \rho_a} \right) (1 - \alpha(t - t_{ref}))$$

$$V_{t_{ref}} = ((O_3 - O_2) / O_1) M_S \left(1 - \frac{\rho_a}{\rho_s} \right) \left(\frac{1}{\rho_w - \rho_a} \right) (1 - \alpha(t - t_{ref}))$$

$$V_x = d_{(F-E)} + \rho_a (V_x - V_s) + [1 - \alpha(t - t_{ref})] + \text{bias/drift} + \text{other corrections}$$

Component	Description	Reference
u _s	Standard uncertainty for standards	Calibration report, divide by k
S _p	Standard uncertainty for the measuring process to determine mass difference between filled and empty container; recorded in volume units on charts to be converted to grams by dividing a Z constant of 1.003; represents all weighing operations	Measurement assurance process; control chart or range chart and d ₂ [*]
u _{bc}	Standard uncertainty for buoyancy correction	
u _{ρ_a}	Standard uncertainty for air density	
u _e	Standard uncertainty for air density equation	CIPM Air density paper (Davis/Giacomo, 1992), 0.0012 mg/cm ³
u _t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.5 °C; SOP 2 (NISTIR 6969)
u _{rh}	Standard uncertainty for relative humidity	HB 143 accuracy guideline, 5 %; SOP 2 (NISTIR 6969)
u _p	Standard uncertainty for pressure	HB 143 accuracy guideline, 65 Pa; SOP 2 (NISTIR 6969)
u _{ρ_s}	Standard uncertainty for density of standard (ρ _s)	Calibration report, e.g., 0.008 g/cm ³
u _{ρ_x}	Standard uncertainty for density of unknown (ρ _x)	Mfg. Estimate, 0.05 g/cm ³
u _{tc}	Standard uncertainty for temperature correction	
u _t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.05 °C

$u_{\text{cce } (\alpha, \beta)}$	Standard uncertainty for cubical coefficient of expansion	NBS Report 10 081, $5 \times 10^{-6} / ^\circ\text{F}$ (0.000009 / $^\circ\text{C}$)
u_{pw}	Standard uncertainty for density of water	
u_e	Standard uncertainty for water density equation and uncorrected systematic error due to air-free vs air-saturated water densities	Metrologia, Patterson/Morris (1994), 7 ppm (0.000007 g/cm ³)
u_t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.1 $^\circ\text{C}$, SOPs
$u_{\text{read men}}$	Standard uncertainty for reading meniscus	(ASTM E694); GMP 3
u_d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
u_{ba}	Standard uncertainty of the balance	Typically not used if s_p is used.
u_{sens}	Standard uncertainty due to sensitivity of balance	OIML R111-1
u_{resol}	Standard uncertainty due to digital resolution of balance	OIML R111-1
$u_{\text{ecc load}}$	Standard uncertainty due to eccentric loading	OIML R111-1
u_{mag}	Standard uncertainty due to magnetism	OIML R111-1
u_o	Standard uncertainty for other factors	

VOLUME – TRANSFER

SOP 16 (FLASKS)

$$V_x = V_{SP} + V_{SB} + V [1 + (\alpha - \beta) (t - t_{ref})] + \text{disparity (bias/drift) + unrelated corrections}$$

Component	Description	Reference
u_s	Standard uncertainty for standards	Calibration report, divide by k
s_p	Standard uncertainty for the process	Measurement assurance process; control chart or range chart and d_2^*
u_{tc}	Standard uncertainty for temperature correction	
u_t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.05 °C; SOPs
$u_{cce (\alpha, \beta)}$	Standard uncertainty for cubical coefficient of expansion	NBS Report 10 081, $5 \times 10^{-6} / ^\circ\text{F}$ (0.000009 / °C)
u_d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
$u_{\text{read men}}$	Standard uncertainty for reading meniscus	(ASTM E694)
u_{drain}	Standard uncertainty due to drain time	
$u_{\text{time piece}}$	Standard uncertainty for time piece	
u_o	Standard uncertainty for other factors	

SOP 18 (GRADUATED NECK)

$$V_x = V_s + \text{disparity (bias/drift) + unrelated corrections}$$

Component	Description	Reference
u_s	Standard uncertainty for standards	Calibration report, divide by k
s_p	Standard uncertainty for the process	Measurement assurance process; control chart or range chart and d_2^*
u_d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
$u_{\text{read men}}$	Standard uncertainty for reading meniscus	(ASTM E694); GMP 3
u_{drain}	Standard uncertainty due to drain time	
$u_{\text{time piece}}$	Standard uncertainty for time piece	
u_o	Standard uncertainty for other factors	

SOP 19 (LARGE NECK)

$$V_x = \frac{\rho_{w_s} \{ V_s [1 + \alpha (t - t_{ref})] \}}{\rho_{w_x} [1 + \beta (t - t_{ref})]} + \text{disparity (bias / drift) + unrelated corrections}$$

Component	Description	Reference
u _s	Standard uncertainty for standards	Calibration report, divide by k
s _p	Standard uncertainty for standards	Measurement assurance process
u _{tc}	Standard uncertainty for temperature correction	
u _t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.05 °C
u _{cce (α,β)}	Standard uncertainty for cubical coefficient of expansion	NBS Report 10 081, 5 x 10 ⁻⁶ /°F (0.000009 / °C)
u _{pw}	Standard uncertainty for density of water	
u _e	Standard uncertainty for water density equation	Metrologia, Patterson/Morris (1994), 7 ppm (0.000007 g/cm ³)
u _t	Standard uncertainty for temperature	HB 143 accuracy guideline, 0.05 °C
u _d	Standard uncertainty for disparity due to drift/bias	Rectangular distribution and reasons, 0.577 d, 0.29 d; SOP 29 (NISTIR 6969)
u _{read men}	Standard uncertainty for reading meniscus	(ASTM E694)
u _{drain}	Standard uncertainty due to drain time	
u _{time piece}	Standard uncertainty for time piece	
u _o	Standard uncertainty for other factors	