GMP No. 9

EQUATIONS FOR METALLIC TAPES

The horizontal straight-line distance, L, of a tape interval can be computed by the following equation for an applied tension, P, and temperature, T, when the tape is supported for N number of equidistant catenary suspensions

$$L = Ls + Ln(P - Ps)/AE + Ln(T - Ts)\alpha - Ln(W \cdot Ln/N \cdot P)^2/24, \qquad (1)$$

where Ls is the calibrated length of the tape interval on a flat surface at Ts and Ps
- Ln is the designated nominal length of the tape interval
- Ps is the standard tension applied to the tape interval for Ls
- Ts is the standard temperature of the tape interval for Ls, 68 °F (20 °C)
- AE is the average cross-sectional area times Young's Modulus of Elasticity
- W is the average weight per unit length of the tape ribbon
- $\alpha$ is the coefficient of thermal expansion of the tape ribbon.

While the tape is supported entirely on a horizontal flat surface, $N = \infty$, the general equation is reduced to

$$L = Ls + Ln(P - Ps)/AE + Ln(T - Ts)\alpha, \qquad (2)$$

and the distance, L, of the tape interval can be set to the designated nominal length, Ln, for determining the tension of accuracy*, Po, while the tape is supported on a flat surface, by writing equation (2) as follows:

$$Ln = Ls + Ln(Po - Ps)/AE + Ln(T - Ts)\alpha,$$

from which

$$Po = Ps + AE(Ln - Ls)/Ln - AE(T - Ts)\alpha \qquad (3)$$

or $$Ps = Po - AE(Ln - Ls)/Ln + AE(T - Ts)\alpha.$$

Substituting the last equation for Ps in the general equation (1), we have

$$L = Ln + Ln(P - Po)/AE - Ln(W \cdot Ln/N \cdot Pc)^2/24,$$

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*Tension of accuracy is defined as that tension which must be applied to the tape interval to produce its designated nominal length at the observed temperature of the tape.

The distance, L, of the tape interval again can be set to the designated nominal length, Ln, for determining the tension of accuracy, Pc, while the tape is supported in catenary suspensions, by writing this equation as follows:

$$Ln = Ln + Ln(Pc - Po)/AE - Ln(W \cdot Ln/N \cdot Pc)^2/24,$$

from which $Pc^2\ (Pc - Po) = AE(W \cdot Ln/N)^2/24$ (4)

or $Pc^2[Pc - Ps - AE(Ln - Ls)/Ln + AE(T - Ts)\alpha] = AE(W \cdot Ln/N)^2/24.$ (5)

The value of Pc can be solved by first determining the right side of the equals sign in equations (4) or (5) then substituting various values for Pc until the left side approaches the right side within the desired limits. If the value is greater than the right side, reduce the value of Pc.